arXiv:0909.4471v2 [hep-ph] 2 Oct 2009

The BFKL-Regge factorization and F_2^b, F_2^c, F_L at HERA: physics implications of nodal properties of the BFKL eigenfunctions

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Abstract

The asymptotic freedom is known to split the leading-log BFKL pomeron into a series of isolated poles in the complex angular momentum plane. One of our earlier findings was that the subleading hard BFKL exchanges decouple from such experimentally important observables as small-x charm, F_2^c, and the longitudinal, F_L, structure functions of the proton at moderately large Q^2. For instance, we predicted precocious BFKL asymptotics of $F_2^c(x,Q^2)$ with intercept of the rightmost BFKL pole $\alpha_{\mathbf{I\!P}}(0)-1=\Delta_{\mathbf{I\!P}}\approx 0.4$. On the other hand, the small-x open beauty photo- and electro-production probes the vacuum exchange for much smaller color dipoles which entails significant subleading vacuum pole corrections to the small-x behavior. In view of the accumulation of the experimental data on small-x F_2^c, F_2^b and F_L we extend our early predictions to the kinematical domain covered by new HERA measurements. Our parameter-free results agree well with the determination of F_2^c, F_L and published H1 results on F_2^b but slightly overshoot the very recent (2008, preliminary) H1 results on F_2^b.

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1. Within the color-dipole (CD) approach to the BFKL pomeron, the flavor independence is a fundamental feature of the dipole cross section, while the QCD pomeron contribution would depend on the interacting particles through the QCD impact factors, calculable in terms of the flavor-dependent color dipole structure of the target and projectile. As noticed by Fadin, Kuraev and Lipatov in 1975 ([1], see also more detailed discussion by Lipatov [2]), incorporation of the asymptotic freedom into the QCD BFKL equation splits the fixed-α_S cut in the complex j-plane into a series of isolated BFKL-Regge poles. Such a spectrum has a far-reaching theoretical and experimental consequences because a contribution of each isolated hard BFKL pole to the scattering amplitudes and/or structure functions (SF) would satisfy a very powerful Regge factorization [3]. The resulting CD BFKL-Regge factorized expansion allows one to relate in a parameter-free fashion SF's of different targets, p, π, γ, γ^* [4, 5, 6] and/or contributions of different flavors to the proton SF [7, 8]. Within the color dipole formulation of the BFKL equation [9] the first analysis of small-x behavior of open charm SF of the proton, F_2^c, in the color dipole formulation of the BFKL equation [9] has been carried out in 1994 [10, 11, 12] with an intriguing result that for moderately large Q^2 it is dominated by the leading hard BFKL pole exchange. Later on this fundamental feature of CD BFKL approach has been related [13] to nodal properties of eigen-functions of subleading hard BFKL-Regge poles [14].

In [14] the latter property of the CD BFKL-Regge factorization was applied and the strength of the subleading hard BFKL corrections and soft-pomeron background to dominant rightmost hard BFKL exchange was quantified. One of the observations of Ref. [14] is that the BFKL-Regge expansion (8) truncated at $m = 2$ appears to be very successful in describing of the proton SF's in a wide range of Q^2. Very recently this phenomenon has been rediscovered in Ref. [15].

In view of the accumulation of the experimental data on small-x F_2^c, F_2^b we extended early predictions to the kinematical domain covered by new HERA measurements. Based on the CD BFKL-Regge factorization we report a parameter-free description of both F_2^c and F_2^b. A specific feature of our CD approach is a decoupling of soft and subleading BFKL singularities at the scale of the open charm production which entails a precocious asymptotic

BFKL behavior of the the structure function F_2^c. Reversing the argument, the open charm excitation by real photons and in DIS gives a particularly clean access to the intercept of the rightmost hard BFKL pole [10]. Here we show how the interplay of leading and subleading vacuum exchanges predicts a rise of the beauty structure function of the proton F_2^b much faster than prescribed by the leading pomeron trajectory (see also the early discussion in Ref. [8]). All our predictions are parameter-free and we find a nice agreement with the published experimental data from H1 Collaboration [16] on the charm and beauty SF of the proton, although the very recent preliminary H1 results on F_2^b [17] are slightly over-predicted. . The longitudinal structure function of the proton F_L is still another observable selective of the dipole size and we report the BFKL-Regge factorization results for F_L. The recent H1 measurements of F_L [18] are consistent with our predictions made in [7] but are too uncertain for any firm conclusions. Taken together, the experimental data on hard structure functions do strongly corroborate our 1994 prediction $\Delta_{\mathbf{I\!P}} \approx 0.4$ for the intercept of the rightmost hard BFKL pole.

2. Within the CD approach to small-x DIS excitation of heavy flavor is described by interactions of $q\bar{q}$ color dipoles in the photon of a predominantly small size $\mathbf{r}$,

$$\frac{4}{Q^2 + 4m_q^2} \lesssim r^2 \lesssim \frac{1}{m_q^2}\,, \tag{1}$$

which makes them an arguably sensitive probe of the short distance properties of the vacuum exchange in QCD in the Regge regime

$$\frac{1}{x} = \frac{W^2 + Q^2}{4m_c^2 + Q^2} \gg 1\,. \tag{2}$$

The CD cross section $\sigma(x, \mathbf{r})$ depends neither on flavor, nor beam, nor target, and the contribution of excitation of the open charm/beauty to photo-absorption cross section is given by the color dipole factorization formula

$$\sigma^c(x, Q^2) = \int dz d^2\mathbf{r} |\Psi_{\gamma^*}^{c\bar{c}}(z, \mathbf{r})|^2 \sigma(x, \mathbf{r})\,. \tag{3}$$

Here $|\Psi_{\gamma^*}^{c\bar{c}}(z, \mathbf{r})|^2$ is the probability to find in the photon the $c\bar{c}$ color dipole with the charmed quark carrying the fraction z ofthe light-cone momentum of the photo [19]. Hereafter we focus on the charm structure function

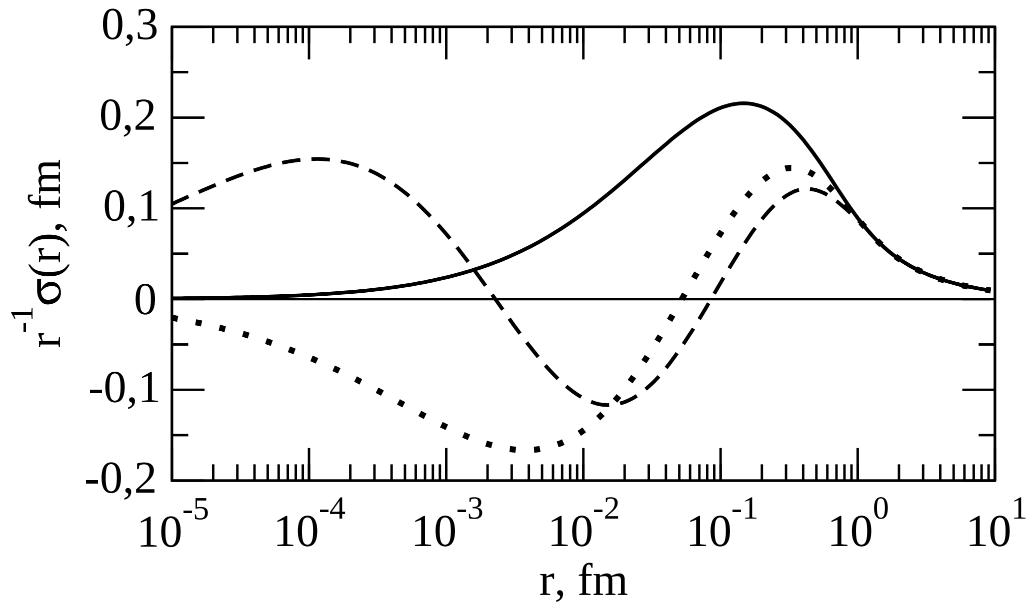

Figure 1: The rightmost σ_0/r and subleading σ_1/r and σ_2/r eigen cross sections as a function of r

are shown.

$$F_2^c(x_{Bj},Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\sigma^c(x,Q^2) = \int \frac{dr^2}{r^2}\frac{\sigma(x,r)}{r^2}W_2(Q^2,m_c^2,r^2)\,. \tag{4}$$

A detailed analysis of the weight function $W_2(Q^2,m_c^2,r^2)$ is found in Refs. [11, 12], we only cite the principal results: (i) at moderate $Q^2 \lesssim 4m_c^2$ the weight function has a peak at a scanning radius $r = r_S \sim 1/m_c$, (ii) at very high Q^2 the peak develops a plateau for dipole sizes in the interval (1), (iii) the contribution from large dipoles is strongly suppressed for heavy flavors. One can say that for moderately large Q^2 excitation of open charm probes (scans) the dipole cross section at a special dipole size r_S (the scanning radius)

$$r_S \sim 1/m_c\,. \tag{5}$$

3. In the Regge region of $\frac{1}{x} \gg 1$ the CD cross section $\sigma(x,r)$ satisfies the CD BFKL equation

$$\frac{\partial\sigma(x,r)}{\partial\log(1/x)} = \mathcal{K}\otimes\sigma(x,r) \tag{6}$$

for the kernel $\mathcal{K}$ of CD approach see Ref. [20]. The solutions with Regge behavior

$$\sigma_m(x,r) = \sigma_m(r)\left(\frac{1}{x}\right)^{\Delta_m} \tag{7}$$

satisfy the eigen-value problem $\mathcal{K}\otimes\sigma_m = \Delta_m\sigma_m(r)$ and the CD BFKL-Regge expansion for the color dipole cross section reads [10, 5]

$$\sigma(x,r) = \sum_{m=0}\sigma_m(r)\left(\frac{x_0}{x}\right)^{\Delta_m}. \tag{8}$$

The practical calculation of $\sigma(x,r)$ requires the boundary condition $\sigma(x_0,r)$ at certain $x_0 \ll 1$. We take for boundary condition at $x = x_0$ the Born approximation, $\sigma(x_0,r) = \sigma_{Born}(r)$, i.e. evaluate dipole-proton scattering via the two-gluon exchange. This leaves the starting point x_0 the sole parameter. The choice $x_0 = 0.03$ met a remarkable phenomenological success [14, 4, 5].

The properties of our CD BFKL equation and the choice of physics motivated boundary condition were discussed in detail elsewhere [11, 12, 13, 14, 4], here we only recapitulate features relevant to the considered problem. Incorporation of asymptotic freedom exacerbates the well known infrared sensitivity of the BFKL equation and infrared regularization by infrared freezing of the running coupling $\alpha_S(r)$ and modeling of confinement of gluons by the finite propagation radius of perturbative gluons R_c need to be invoked.

The leading eigen-function $\sigma_0(r) \equiv \sigma_{\mathbf{I\!P}}(r)$ for ground state i.e., for the rightmost hard BFKL pole is node free. The subleading eigen-function for the excited state $\sigma_m(r)$ has m nodes (see Fig. 1). We solve for $\sigma_m(r)$ numerically [14, 4] (for the semi-classical analysis see the paper of Lipatov of Ref. [2]. The so found intercepts (binding energies) follow to a good approximation the law of Lipatov, $\Delta_m = \Delta_0/(m+1)$. For the preferred $R_c = 0.27\,\mathrm{fm}$ as chosen in 1994 in Refs. [12, 11] and supported by the analysis [21] of lattice QCD data we find $\Delta_0 \equiv \Delta_{\mathbf{I\!P}} = 0.4$. The node of $\sigma_1(r)$ is located at $r = r_1 \simeq 0.056\,\mathrm{fm}$, for larger m the rightmost node moves to a somewhat larger $r = r_1 \sim 0.1\,\mathrm{fm}$. The second node of eigen-functions with $m = 2,3$ is located at $r_2 \sim 3\cdot 10^{-3}$ fm which corresponds to the momentum transfer scale $Q^2 = 1/r_2^2 = 5\cdot 10^3$ GeV^2. The third node of $\sigma_3(r)$ is located at r beyond the reach of any feasible DIS experiments. It has been found [14] that the BFKL-Regge expansion (8) truncated at $m = 2$ appears to be very successful in describing of the proton SF's at $Q^2 \lesssim 200$ GeV^2. However, at higher Q^2 and moderately small $x \sim x_0 = 0.03$ the background of the CD BFKL solutions with smaller intercepts ($\Delta_m < 0.1$) should be taken into account (see below).

Now comes the crucial observation that numerically $r_1 \sim r_S$. Consequently, in the calcu-

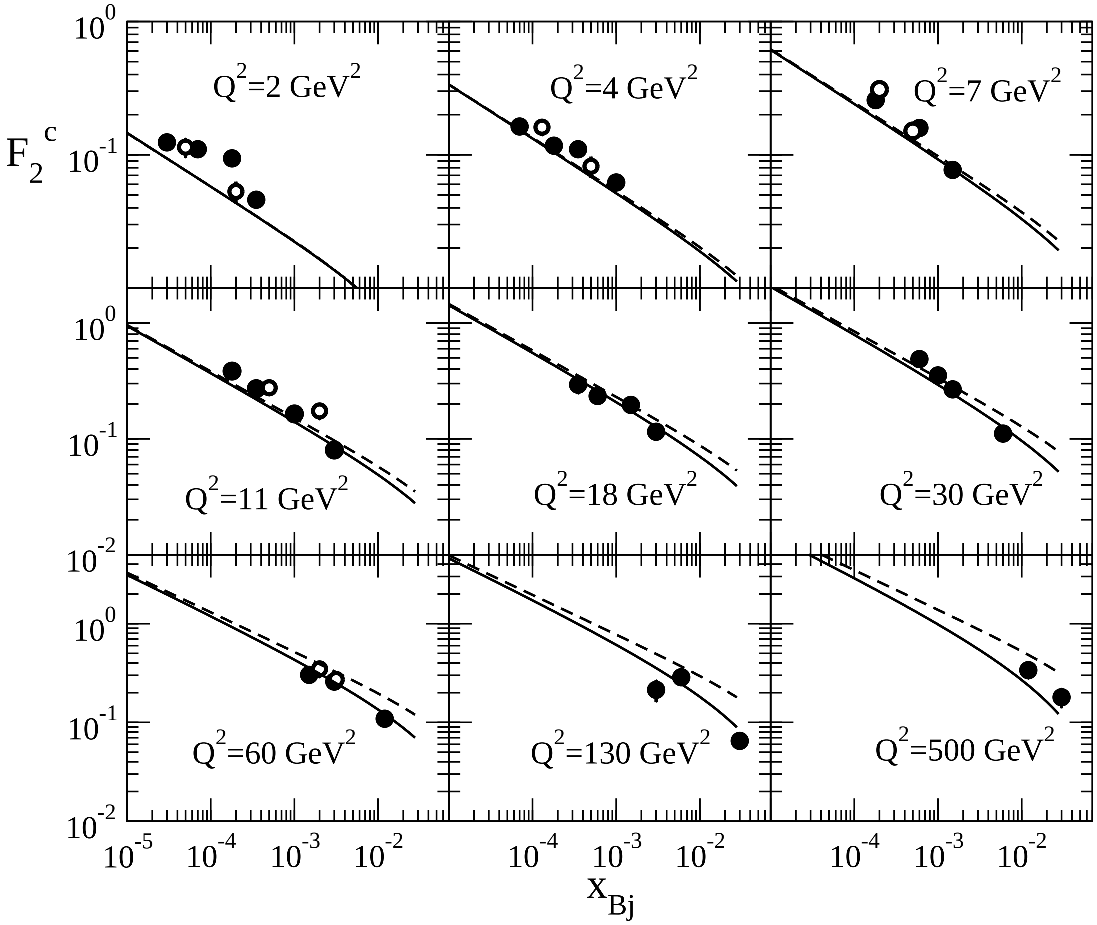

Figure 2: The prediction from CD BFKL-Regge factorization for the charm structure function of the proton $F_2^c(x, Q^2)$ as a function of the Bjorken variable x_{Bj} is compared with the experimental data from H1 Collaboration [16]. The solid curve represents the result of the complete CD BFKL-Regge expansion, the contribution of the rightmost hard BFKL pole with $\Delta_{\mathbf{IP}} = 0.4$ is shown by the dashed line

4. Because the probability to find large color dipoles in the photon decreases rapidly with the quark mass, the contribution from soft-pomeron exchange to open charm excitation is very small down to $Q^2 = 0$. As we discussed elsewhere [5, 7], for still higher solutions, $m \geq 3$, all intercepts are very small anyway, $\Delta_m \ll \Delta_0$, For this reason, on the purpose of practical phenomenology, we truncate the expansion (8) at $m = 3$ lumping in the term $m = 3$ the contributions of still higher singularities with $m \geq 3$. The term $m = 3$ is endowed with the effective intercept $\Delta_3 = 0.06$ and is presented in Ref. [7] in its analytical form.

We comment first on the results on F_2^c. The solid curve in Fig. 2 is the result of the complete CD BFKL-Regge expansion. The dashed curve shows the pure rightmost hard BFKL pomeron contribution, in the Leading Hard Approximation (LHA). There is a strong cancellation between soft and subleading contributions with $m = 1$ and $m = 3$. Consequently, for this dynamical reason in this region of $Q^2 \lesssim 10$ GeV^2 we have an effective one-pole picture and LHA gives reasonable description of F_2^c.

In agreement with the nodal structure of subleading eigen-SF's discussed in Refs. [5, 7], the LHA over-predicts slightly F_2^c at $Q^2 \gtrsim 30$ GeV^2. Here the negative valued subleading hard BFKL exchanges overtake the soft-pomeron exchange and the background from subleading hard BFKL exchanges becomes substantial at $Q^2 \gtrsim 30$ GeV^2 and would even dominate F_2^c at $Q^2 \gtrsim 200$ GeV^2 and $x \gtrsim 10^{-2}$. In this region of Q^2 the soft-pomeron exchange is numerically negligible. The aforementioned soft-subleading cancellations at $Q^2 \lesssim 20$ GeV^2 become less accurate at smaller x, but here both soft and subleading hard BFKL exchanges become Regge suppressed, because proportional to $x^{\Delta_{\mathbf{I\!P}}}$, $x^{\Delta_{\mathbf{I\!P}}/2}$, respectively.

In Fig. 2 we compare our CD BFKL-Regge predictions to the recent experimental data from the H1 Collaboration [16] and find a very good agreement between the theory and experiment which lends support to our 1994 evaluation $\Delta_{\mathbf{I\!P}} = 0.4$. The negative valued contribution from the subleading hard BFKL exchange is important for bringing the theory to agreement with the experiment at large Q^2. For an alternative interpretation of heavy flavor production see Refs. [22, 23, 24] and references therein.

5. The characteristic feature of the QCD pomeron dynamics at distances $\sim m_b^{-1}$ is the large negative valued contribution to F_2^b, coming from subleading BFKL singularities, see Fig. 1 and Ref. [8]. Consequences of this observation for the exponent of the energy dependence of the structure function

$$F_2^b \propto \left(\frac{x_0}{x}\right)^{\Delta_{\rm eff}} \tag{9}$$

are quite interesting. In terms of the ratio $r_m = \sigma_m/\sigma_0$ (see Fig. 1) the exponent $\Delta_{\rm eff}$ reads (m=1,2,3,soft)[8]

$$\Delta_{\rm eff} = \Delta_0 \left[1 - \sum_{m=1} r_m (1 - \Delta_m/\Delta_0)(x_0/x)^{\Delta_m - \Delta_0}\right] \tag{10}$$

Coefficients r_m in Eq. (10) depend on r. They are negative on the left from the rightmost node

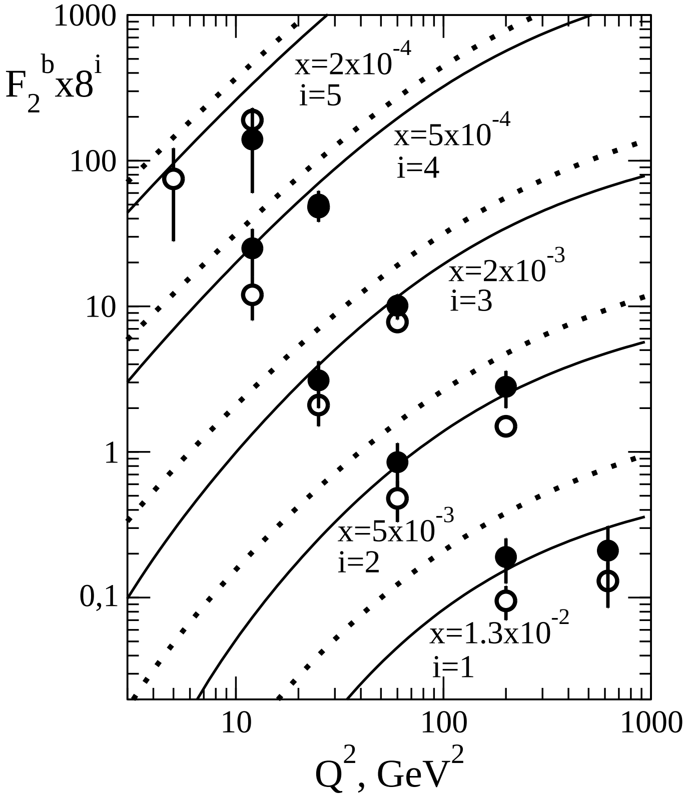

Figure 3: The predictions from CD BFKL-Regge factorization for the beauty structure function $F_2^b(x,Q^2)$, compared with data o Refs. [16] (full circles) and [17] (open circles) are shown. The solid curve is the result of the complete CD BFKL-Regge expansion, the contribution of the rightmost hard BFKL pole with $\Delta_{\mathbf{IP}} = 0.4$ is repesented by dotted curves.

(Fig. 1) and positive on the right. Because for $r \sim m_b^{-1}$ all r_m are negative, except $r_{\rm soft}(0) > 0$ [8], at HERA energies the effective intercept $\Delta_{\rm eff} \equiv \Delta_{\rm beauty}$ overshoots the asymptotic value $\Delta_{\mathbf{IP}} \equiv \Delta_0 = 0.4$. At still higher collision energies both the soft and subleading hard BFKL exchanges become rapidly Regge suppressed and we expect $\Delta_{\rm eff}$ to decrease down to $\Delta_{\mathbf{IP}}$ [8]. This must be constructed to an aforementioned positive valued subleading BFKL and soft terms in the CD BFKL-Regge expansion for light flavor SF's of the proton (see [5] for more details), which lowers the pre-asymptotic pomeron intercept in photoproduction of light flavors. Hence the CD prediction of the hierarchy of pre-asymptotic intercepts [8] is

$$\Delta_{\rm beauty} > \Delta_{\rm charm} > \Delta_{\rm light}\,. \qquad (11)$$

In Fig. 3 we presented our predictions for the beauty structure function. The solid curve corresponds to the complete expansion (8) while the dotted curve is the LHA. In agreement

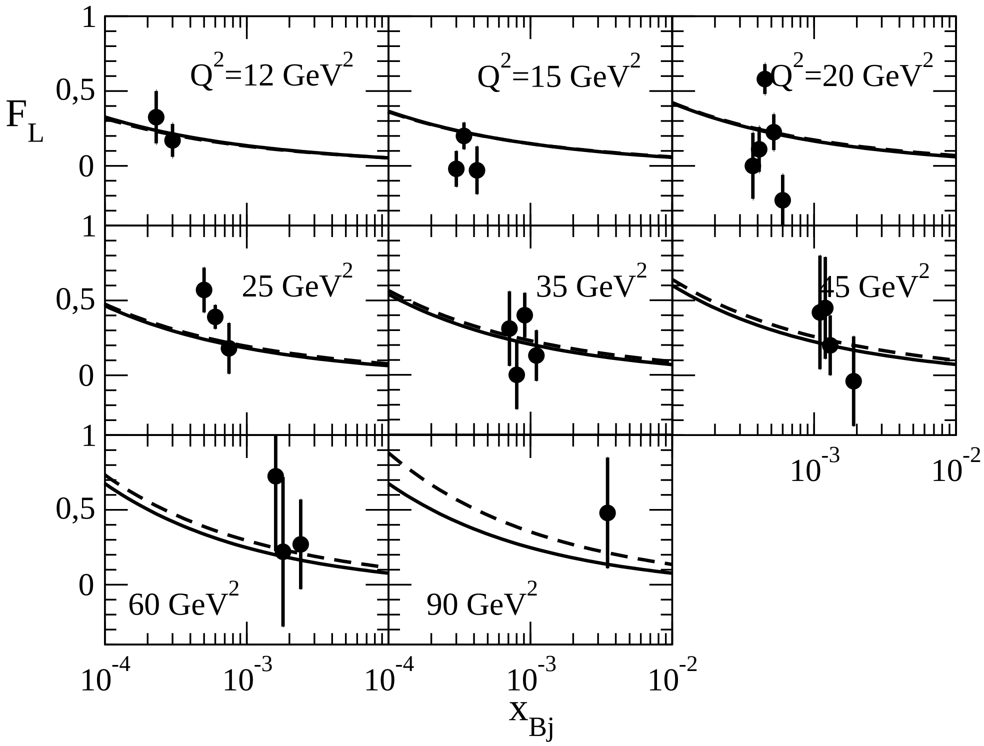

Figure 4: The prediction from CD BFKL-Regge factorization for the longitudinal structure function of the proton $F_L(x, Q^2)$ as a function of the Bjorken variable x_{Bj} is shown. The solid curve is the result of the complete CD BFKL-Regge expansion, the contribution of the rightmost hard BFKL pole with $\Delta_{\mathbf{IP}} = 0.4$ is represented by dashed line. Data points ar from Ref. [18]

with the nodal structure of subleading eigen-SF's the latter over-predicts F_2^b significantly because the negative valued contribution from subleading hard BFKL exchanges overtakes the soft-pomeron exchange and the background from subleading hard BFKL exchanges is substantial for all Q^2 values [8]. Our predictions for the beauty SF agree well with the determination of F_2^b by the H1 collaboration published in 2006 [16] (full circles) but slightly overshoot smaller values of F_2^b from the very recent preliminary H1 results on F_2^b reported in 2008 [17] (open circles).

6. The cross section of diffractive (elastic) $\Upsilon(1S)$ meson photoproduction has been measured at HERA [25]. Quarks in Υ meson are nonrelativistic and for real photons the $b\bar{b}$ CD wave function of the photon $|\gamma\rangle is proportional to m_b K_0(m_b r)$, where $K_0(x)$ is the Bessel-MacDonald function [26]. The forward $\gamma \to \Upsilon$ transition matrix element $\langle\Upsilon|\sigma_n(r)|\gamma\rangle$ is

controlled by the product $\sigma_0(r)K_0(m_b r)$ [26] and the amplitude of elastic $\Upsilon(1S)$ photoproduction is dominated by the contribution from the dipole sizes $r \sim r_\Upsilon = A/m_\Upsilon$ with $A \approx 5$. For a recent review on diffractive vector mesons see Ref. [27].

The crucial observation is that at distances $r \sim r_\Upsilon$ cancellations between soft and subleading contributions to the elastic photoproduction cross section result in the exponent Δ in

$$\frac{d\sigma(\gamma p \to \Upsilon p)}{dt}|_{t=0} \propto W^{4\Delta} \tag{12}$$

which is very close to $\Delta_{\mathbf{IP}}$, $\Delta = 0.38$ [8, 28]. This observation appears to be in agreement with the cross section rise observed by ZEUS and H collaborations [25].

7. It has been demonstrated in Ref. [11] that the longitudinal structure function $F_L(x, Q^2)$ emerges as local probe of the dipole cross section at $r^2 \simeq 11/Q^2$. The subleading CD BFKL cross sections have their rightmost node at $r_1 \sim 0.05 - 0.1$ fm. Therefore, one can zoom at the leading CD BFKL pole contribution and measure the pomeron intercept $\Delta_{\mathbf{IP}}$ from the x-dependence of $F_L(x, Q^2)$ at $Q^2 \sim 10 - 30$ GeV^2. The aforementioned soft-subleading cancellation is nearly exact at $Q^2 \sim 10 - 30$ GeV^2 and we predict a leading hard pole dominance in this region, as one can see from see Fig. 4), where comparison with the very recent H1 data [18] is presented. We predicted the correct magnitude of $F_L(x, Q^2)$, although the experimental data do not allow to draw conclusions on the x-dependence.

8. A simple note in passing. Okun and Pomeranchuk argued that for the members of the same isotopic multiplet the strong interaction cross sections would have an identical high energy behavior [29]. Compare the total cross section of the charged and neutral components of isotriplets of mesons like the ρ-mesons or pions on an electrically neutral target like a neutron. Arguably, for such a target the electromagnetic breaking of the Okun-Pomeranchuk theorem will be dominated by the electromagnetic lifting of the degeneracy of sizes of the charged and neutral ρ's. The strength of the Coulomb interaction in the charge and neutral mesons is proportional to $e_u e_d$ and $-(e_u^2 + e_d^2)/2$, respectively, the net difference being proportional to $(e_u + e_d)^2$. Consequently, the difference of the radii mean squared can be estimated as $\sim \alpha_{em}(e_u + e_d)^2 \langle r^2 \rangle$, what would entail

$$\frac{\sigma_\pm - \sigma_o}{\sigma_\pm + \sigma_o} \sim \alpha_{em}(e_u + e_d)^2 \ . \tag{13}$$

9. The color dipole approach to the BFKL dynamics predicts uniquely a decoupling of subleading hard BFKL exchanges from open charm SF of the proton at $Q^2 \lesssim 20\,\mathrm{GeV}^2$ and from F_L at $Q^2 \simeq 20\,\mathrm{GeV}^2$. This decoupling is due to a dynamical cancellation between contributions of different subleading hard BFKL poles and leaves us with an effective soft+rightmost hard BFKL two-pole approximation with intercept of the soft pomeron $\Delta_{\mathrm{soft}} = 0$. We predict strong cancellation between the soft-pomeron and subleading hard BFKL contribution to F_2^c in the experimentally interesting region of $Q^2 \lesssim 20$ GeV^2, in which F_2^c is dominated entirely by the contribution from the rightmost hard BFKL pole. This makes open charm in DIS at $Q^2 \lesssim 20$ GeV^2 a unique handle on the intercept of the rightmost hard BFKL exchange. Similar hard BFKL pole dominance holds for $F_L(x, Q^2)$.

High-energy open beauty photoproduction probes the behavior $r \sim 1/m_b$ for the color dipole size and picks up a significant contribution from the subleading BFKL poles. This makes $\sigma^{b\bar{b}}(W)$ to grow much faster than it is prescribed by the leading BFKL pole with an intercept $\alpha_{\mathbf{IP}}(0) - 1 = \Delta_{\mathbf{IP}} = 0.4$. Our calculations within the color dipole BFKL model are in agreement with the recent determination of $\sigma^{b\bar{b}}(W)$ by the H1 collaboration. The comparative analysis of diffractive photoproduction of beauty, charm and light quarks exhibits the hierarchy of pre-asymptotic pomeron intercepts which follows the hierarchy of corresponding hardness scales. We comment on the phenomenon of decoupling of soft and subleading BFKL singularities at the scale of elastic $\Upsilon(1S)$ -photoproduction which results in precocious color dipole BFKL asymptotics of the process $\gamma p \to \Upsilon p$. The agreement with the presently available experimental data on open charm/beauty in DIS confirm the CD BFKL prediction of the intercept $\Delta_{\mathbf{IP}} = 0.4$ for the rightmost hard BFKL-Regge pole.

Acknowledgments: V.R. Z. thanks the Dipartimento di Fisica dell'Università della Calabria and the Istituto Nazionale di Fisica Nucleare - gruppo collegato di Cosenza for their warm hospitality while a part of this work was done. The work was supported in part by the Ministero Italiano dell'Istruzione, dell'Università e della Ricerca and by the RFBR grants 07-02-00021 and 09-02-00732.

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